NAG Toolbox for MATLAB

f04mc

1 Purpose

f04mc computes the approximate solution of a system of real linear equations with multiple right-hand sides, $AX = B$, where A is a symmetric positive-definite variable-bandwidth matrix, which has previously been factorized by f01mc. Related systems may also be solved.

2 Syntax

```
[x, ifail] = f04mc(al, d, nrow, b, iselct, 'n', n, 'lal', lal, 'ir', ir)
```

3 Description

The normal use of this function is the solution of the systems $AX = B$, following a call of f01mc to determine the Cholesky factorization $A = LDL^{\mathrm{T}}$ of the symmetric positive-definite variable-bandwidth matrix A.

However, the function may be used to solve any one of the following systems of linear algebraic equations:

1. $LDL^{\mathrm{T}}X = B$ (usual system),
2. $LDX = B$ (lower triangular system),
3. $DL^{\mathrm{T}}X = B$ (upper triangular system),
4. $LL^{\mathrm{T}}X = B$
5. $LX = B$ (unit lower triangular system),
6. $L^{\mathrm{T}}X = B$ (unit upper triangular system).

L denotes a unit lower triangular variable-bandwidth matrix of order n, D a diagonal matrix of order n, and B a set of right-hand sides.

The matrix L is represented by the elements lying within its **envelope**, i.e., between the first nonzero of each row and the diagonal (see Section 9 for an example). The width **nrow**(i) of the ith row is the number of elements between the first nonzero element and the element on the diagonal inclusive.

4 References

Wilkinson J H and Reinsch C 1971 *Handbook for Automatic Computation II, Linear Algebra* Springer–Verlag

5 Parameters

5.1 Compulsory Input Parameters

1: **al**(**lal**) – **double array**

The elements within the envelope of the lower triangular matrix L, taken in row by row order, as returned by f01mc. The unit diagonal elements of L must be stored explicitly.

2: **d**$(*)$ – **double array**

Note: the dimension of the array **d** must be at least 1 if **iselct** ≥ 4, and at least **n** otherwise.

The diagonal elements of the diagonal matrix D. **d** is not referenced if **iselct** ≥ 4.

3: **nrow(n) – int32 array**

nrow(i) must contain the width of row i of L, i.e., the number of elements between the first (leftmost) nonzero element and the element on the diagonal, inclusive.

Constraint: $1 \leq$ **nrow**$(i) \leq i$.

4: **b(ldb,ir) – double array**

ldb, the first dimension of the array, must be at least **n**.

The n by r right-hand side matrix B. See also Section 8.

5: **iselct – int32 scalar**

Must specify the type of system to be solved, as follows:

iselct $= 1$

Solve $LDL^{\mathrm{T}}X = B$.

iselct $= 2$

Solve $LDX = B$.

iselct $= 3$

Solve $DL^{\mathrm{T}}X = B$.

iselct $= 4$

Solve $LL^{\mathrm{T}}X = B$.

iselct $= 5$

Solve $LX = B$.

iselct $= 6$

Solve $L^{\mathrm{T}}X = B$.

Constraint: **iselct** $= 1, 2, 3, 4, 5$ or 6.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the array **nrow**.

n, the order of the matrix L.

Constraint: **n** ≥ 1.

2: **lal – int32 scalar**

Default: The dimension of the array **al**.

Constraint: **lal** $\geq$ **nrow**(1) + **nrow**$(2) + \ldots +$ **nrow**(n).

3: **ir – int32 scalar**

Default: The dimension of the arrays **b**, **x**. (An error is raised if these dimensions are not equal.)

r, the number of right-hand sides.

Constraint: **ir** ≥ 1.

5.3 Input Parameters Omitted from the MATLAB Interface

ldb, ldx

5.4 Output Parameters

1: **x(ldx,ir) – double array**

The n by r solution matrix X. See also Section 8.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, $\mathbf{n} < 1$,
or for some i, $\mathbf{nrow}(i) < 1$ or $\mathbf{nrow}(i) > i$,
or $\mathbf{lal} < \mathbf{nrow}(1) + \mathbf{nrow}(2) + \cdots + \mathbf{nrow}(\mathbf{n})$.

ifail $= 2$

On entry, $\mathbf{ir} < 1$,
or $\mathbf{ldb} < \mathbf{n}$,
or $\mathbf{ldx} < \mathbf{n}$.

ifail $= 3$

On entry, $\mathbf{iselct} < 1$,
or $\mathbf{iselct} > 6$.

ifail $= 4$

The diagonal matrix D is singular, i.e., at least one of the elements of **d** is zero. This can only occur if $\mathbf{iselct} \leq 3$.

ifail $= 5$

At least one of the diagonal elements of L is not equal to unity.

7 Accuracy

The usual backward error analysis of the solution of triangular system applies: each computed solution vector is exact for slightly perturbed matrices L and D, as appropriate (see pages 25–27 and 54–55 of Wilkinson and Reinsch 1971).

8 Further Comments

The time taken by f04mc is approximately proportional to pr, where $p = \mathbf{nrow}(1) + \mathbf{nrow}(2) + \cdots + \mathbf{nrow}(n)$.

9 Example

```
al = [1;
      2;
      1;
      3;
      1;
      1;
      5;
      4;
```

```
     1.5;
     0.5;
     1;
     1.5;
     5;
     1];
d = [1;
     1;
     4;
     16;
     1;
     16];
nrow = [int32(1);
     int32(2);
     int32(2);
     int32(1);
     int32(5);
     int32(3)];
b = [6, -10;
     15, -21;
     11, -3;
     0, 24;
     51, -39;
     46, 67];
iselct = int32(1);
[x, ifail] = f04mc(al, d, nrow, b, iselct)
```

```
x =
    -3     4
     2    -2
    -1     3
    -2     1
     1    -2
     1     1
ifail =
           0
```